Average of the values of x

$$\overline{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Average of the values of x^2

$$\overline{x^2} = \frac{\sum_{i=1}^{N} {x_i}^2}{N}$$

Definition of variance and what it is for large values of N

$$\sigma^2 = E\left[X^2\right] - E[X]^2 \approx \overline{x^2} - \overline{x}^2$$

Unbiased Variance

$$\widehat{\sigma^2} = \frac{\sum_{i=1}^{N} (x_i - \overline{x})^2}{N-1}$$

$$\widehat{\sigma^2} = \frac{\sum_{i=1}^{N} {x_i}^2}{N-1} - \frac{\sum_{i=1}^{N} 2x_i\overline{x}}{N-1} + \frac{\sum_{i=1}^{N} \overline{x}^2}{N-1}$$

$$\widehat{\sigma^2} = \frac{1}{N-1}\left(\sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} 2x_i\overline{x} + \sum_{i=1}^{N} \overline{x}^2\right)$$

$$\widehat{\sigma^2} = \frac{1}{N-1}\left(N\overline{x^2} - 2N\overline{x}\,\overline{x} + N\overline{x}^2\right)$$

$$\boxed{\widehat{\sigma^2} = \frac{N}{N-1}\left(\overline{x^2} - \overline{x}^2\right) \qquad \widehat{\sigma^2} = \frac{N}{N-1}\sigma^2}$$

Note the unbiased variance is an overestimate version of a variance.

Explanation of Expected values vs average

n is the number of samples, then μ is the average of the samples x_i, where i = 1,2,…n

$$\mu = (x_1 + x_2 + \cdots + x_n)/n$$

Suppose there exists samples x_i and x_j where j and i are integers such that x_i and x_j are equal. Then

$$\mu = (x_1(\#of\ x = x_1) + x_2(\#of\ x = x_2) + \cdots + x_k(\#of\ x = x_k))/n$$

$$\mu = (x_1(\#of\ x = x_1)/n + x_2(\#of\ x = x_2)/n + \cdots + x_k(\#of\ x = x_k)/n)$$

$$\mu = (x_1(\%\,x = x_1) + x_2\,(\%x = x_2) + \cdots + x_k(\%x = x_k))$$

$$\mu = \sum_{i=1}^{n} (\%x = x_i)x_i$$

where n is large (laws of large numbers) it will be = instead

$$\mu = \sum_{i=1}^{n} x_i(\%x = x_i) \approx E[X] = \int x f_x(x)$$

Where $f_x(x)$ is the pdf (probability distribution function) of x

Moving Average

We would like to find the moving average, an equation that will find the average of a number of previous samples. Due to calculation time, and the amount of moving averages, we would like this equation to be memoryless, or the need to store as little amount of values possible. Imagine a queue

$x_0 \leftarrow$	x_1	x_2	x_3	...	x_n	$\leftarrow x_{n+1}$

Above we would like to analyze the data within the block, where x_0 is the point being queued out and x_{n+1} is a future point being queued in. We can use the equation above to find the average in the block.

$$\bar{x} = \frac{\sum_{i=1}^{N} x_i}{N}$$

Now the average after popping and queueing in the next sample

$x_1 \leftarrow$	x_2	x_3	x_4	...	x_{n+1}	$\leftarrow x_{n+2}$

The new average would just be

$$\overline{x_{t+1}} = \frac{n\bar{x}_t - x_1 + x_{n+1}}{n}$$

For the sake of keeping memory, we can't keep track of all samples of x so we are going to make a number of assumptions. Hopefully you are sampling fast enough so you can keep track of the signal if the distribution of the samples is not stationary. The sample we are popping out is approximately/most likely the same as the current average. Substituting that we get,

$$\widehat{x_{t+1}} = \frac{n\overline{x_t} - \overline{x_t} + x_{n+1}}{n}$$

$$\boxed{\widehat{x_{t+1}} = \frac{(n-1)}{n}\overline{x_t} + \frac{x_{n+1}}{n}}$$

Which means we only need to keep track of the current average and the incoming sample to find the new average. This follows the actual average and if the system is in steady state, the moving average will eventually equal the actual average. The value of n does affect how fast the moving average equal the actual average, (Example of different values of n affecting moving average here under the wind sensor page for real time process {Code is the same except without the absolute value}
https://wiki.scel-hawaii.org/doku.php?id=wind_sensor:wind_sensor_documentation#implementing_code).

Moving variance.

$x_0 \leftarrow$	x_1	x_2	x_3	...	x_n	$\leftarrow x_{n+1}$

Back to the original picture, we can find the variance of the set within the block with the equation above.

$$\widehat{\sigma^2} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{N}}(x_i - \overline{x})^2}{N-1}$$

Now the average after popping and queueing in the next sample

$x_1 \leftarrow$	x_2	x_3	x_4	…	x_{n+1}	$\leftarrow x_{n+2}$

The new variance would be

$$\widehat{\sigma^2_{t+1}} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{N}}(x_i - \overline{x_t})^2 - (x_1 - \overline{x_t})^2 + (x_{n+1} - \overline{x_{t+1}})^2}{N-1}$$

Just like before we are going to assume the value x that is being popped out is approximately the moving average and the moving average doesn't change that much between the sample so $\overline{x_{t+1}} \approx \overline{x_t}$

$$\widehat{\widehat{\sigma^2_{t+1}}} = \frac{\sum_{\mathrm{i=1}}^{\mathrm{N}}(x_i - \overline{x_t})^2 - (\overline{x_t} - \overline{x_t})^2 + (x_{n+1} - \overline{x_t})^2}{N-1}$$

$$\widehat{\widehat{\sigma^2_{t+1}}} = \widehat{\sigma^2_t} - \frac{(\overline{x_t} - \overline{x_t})^2 + (x_{n+1} - \overline{x_t})^2}{N-1}$$

This is now kind of in the form of a moving average except instead of x_n it is $(x_n - \overline{x_t})^2$ and instead of N, it is N-1. So

$$\boxed{\widehat{\widehat{\sigma^2_{t+1}}} = \frac{N-1}{N-2}\widehat{\sigma^2_t} + \frac{(x_{n+1} - \overline{x_t})^2}{N-2}}$$

Another method is for finding the moving variance is with this equation

$$\widehat{\sigma^2} = \frac{N}{N-1}(\overline{x^2} - \overline{x}^2)$$

Using the moving average equations (meaning instead of keeping track of variances and averages, we can keep track of the moving average of the first and second moment and calculate the unbiased variance like that)

$$\boxed{\widehat{\widehat{\sigma^2_t}} = \frac{N}{N-1}(\overline{x_t^2} - \overline{x_t}^2)}$$

Where

$$\boxed{\widehat{\overline{x^2_{t+1}}} = \frac{(n-1)}{n}\overline{x^2_t} + \frac{x^2_{n+1}}{n}}$$